Surface Area Measure Program Information

1. Contributors and history.

An algorithm for the purpose of reconstructing a convex polytope from its surface area measure (effectively, the areas and outer unit normal vectors of its facets) was first published by Little [5]. Several groups of researchers considered the problem after Little; see the introduction of [3] for more information. In particular, motivated by computer vision, Lemordant et al. [4] modified Little's algorithm. Their approach is used in the algorithm published by Gardner and Milanfar [1, Section A.4] (see also [2]) under the name Algorithm MinkData. Implementation of this algorithm was done independently of earlier implementations and was carried out initially with the assistance of Western Washington University (WWU) undergraduate student Chris Street. From 2003 to 2006, improvements were made by WWU undergraduate students Chris Eastman, Thomas Riehle, and Greg Richardson. The original web page GUI was designed by WWU undergraduate student Dale Jennings in 2010 and subsequently modified by WWU undergraduate students Roberto Vergaray, Ian Fisk, and Elliott Skomski. Both the mathematics and implementation were supported by National Science Foundation grants DMS-9802388, DMS-0203527, DMS-0603307, DMS-1103612, and DMS-1402929.

2. How the program works.

A detailed description can be found in [1, Section A.4]. The key underlying theoretical result is Minkowski's existence theorem [1, Theorem A.3.2] and its proof, but the Brunn-Minkowski inequality [1, Section B.2] also plays a role. Algorithm MinkData involves solving a constrained nonlinear optimization problem, but the saving feature is that the objective function is convex and the constraints are linear. The solution to this problem yields a description of the approximating convex polytope (in the implementation, a polyhedron since $n = 3$) in terms of the planes containing its facets, its so-called $\mathcal{H}$-representation. This has to be converted to a list of its vertices, i.e., its $\mathcal{V}$-representation, after which Matlab's convex hull function and graphics package provide a picture.

References

[1] R. J. Gardner, *Geometric Tomography*, second edition, Cambridge University Press, New York, 2006.

[2] R. J. Gardner and Peyman Milanfar, Shape reconstruction from brightness functions, in: *Proceedings of SPIE Conference on Advanced Signal Processing Algorithms, Architectures, and Implementations X*, San Diego, CA, 2001, Proceedings of SPIE **4474**, pp. 234–245.

[3] R. J. Gardner and P. Milanfar, Reconstruction of convex bodies from brightness functions, *Discrete Comput. Geom.* **29** (2003), 279–303.

[4] J. Lemordant, P. D. Tao, and H. Zouaki, Modélisation et optimisation numérique pour la reconstruction d'un polyèdre à partir de son image gaussienne généralisée, *RAIRO Modél. Math. Anal. Numér.* **27** (1993), 349–374.

[5] J. J. Little, *An iterative method for reconstructing convex polyhedra from extended Gaussian images*, in: Proc. AAAI, National Conf. Artificial Intelligence (Washington, D.C., 1983), pp. 247–250.